Complex Number 4

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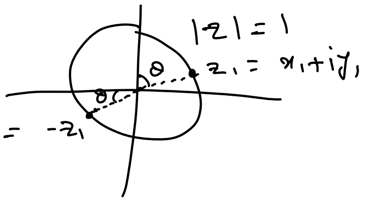
$|z| = 1 \Rightarrow z\bar{z} = 1 \Rightarrow (x+iy)(x-iy) = x^2+y^2 = 1$

$|z|^2 = 1$



$z_1(-z_1) = -(z_1)^2 = -(x_1+iy_1)^2$

$= -x_1^2 - y_1^2 - 2xy_1 i = (-1-2xyi)$

$= -z_1^2$

$\frac{1}{z_1} = \frac{1}{x_1+iy_1} = \frac{x_1-iy}{(x_1+iy_1)(x_1-iy_1)} = \frac{x_1-iy_1}{x_1^2+y_1^2} = x_1-iy_1 = \bar{z}_1$

$\bar{z}_1 = \frac{1}{z_1}$

$a \in \mathbb{R}^+$ Then minimum value of $a + \frac{1}{a}$ $\qquad$ AM $\geq$ GM

↳ positive

$\frac{a+\frac{1}{a}}{2} \geq \sqrt{a\frac{1}{a}}$ $\qquad$ For $a = 1$, $a + \frac{1}{a} = 2$

$(a+\frac{1}{a}) \geq 2$ ↳ Min value

$a \in \mathbb{R}^-$. Then max value of $a + \frac{1}{a}$ $\qquad$ AM $\geq$ GM

$-$AM $\leq$ $-$GM

$a = -b \quad b \in \mathbb{R}^+$ $\qquad -b - \frac{1}{b}$

$\boxed{\frac{b+\frac{1}{b}}{2} \geq \sqrt{b\frac{1}{b}}} \Rightarrow -\frac{(b+\frac{1}{b})}{2} \leq -\sqrt{b\frac{1}{b}} = -1$

$-(b+\frac{1}{b}) \leq -2$

$a + \frac{1}{a} \leq -2 \rightarrow$ Max value

$|z| = 1$

$z\bar{z} = 1$

$\frac{1}{z} = \bar{z}$

$\left|z + \frac{1}{z}\right|$ max value and min value

$\leq |z| + \left|\frac{1}{z}\right| = 2 \rightarrow$ max value

No. of equation $\geq$ No. of variables for a solution

$z = x+iy \Rightarrow \left|x+iy + \frac{1}{x+iy}\right| = \left|z + \frac{1}{z}\right| = |z+\bar{z}|$

$= |2x| \geq 0 \rightarrow$ minimum value

HW :- Solve this using only x & y and algebraic manipulation.
